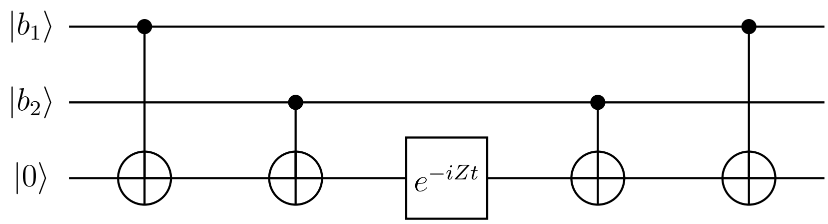
33-658 Quantum Computing and Quantum Information Homework 10

1. Quantum circuit magic (phase kickback)

a) Evaluate the final state for the circuit below. Can you interpret your result as the time evolution operator for a Hamiltonian acting on bits 1 and 2?



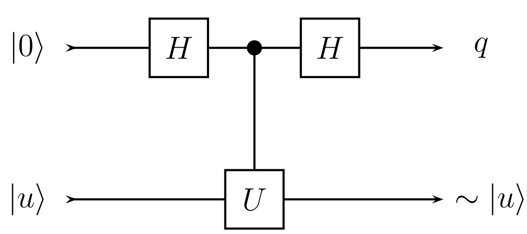
b) Why was it necessary to perform the second set of `CNOT`'s following the e^{-iZt}?

c) Taking inspiration from (a), propose a circuit to evaluate the time evolution operator for the Hamiltonian $H = X_1 \otimes Y_2$.

2. Trotterization

$H = \Omega_z Z + \Omega_x X$ is a typical qubit Hamiltonian with Ω_x and Ω_z fixed parameters. Set up a Trotter decomposition for the time evolution operator $U(t)$. Approximately how many time slices, M, should you apply for a given time t?

3. In the Kitaev algorithm (see circuit below) $|u\rangle$ is an eigenstate of a unitary operator U with eigenvalue $e^{2\pi i\varphi}$. Show that measurements of the top qubit yields $q = 0$ with probability $P(q = 0) = \cos^2(\pi\varphi)$. Since the state $|u\rangle$ is unaffected by the measurement of q, up to a complex phase factor, it can be reused and sent through subsequent operations U^k. Explain how to use repeated applications of U^k followed by measurement of q to obtain bits of the value of $P(q = 0)$, and hence φ.



4. Design a quantum circuit to map the single qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ to the three qubit state $|\Psi\rangle = \alpha|000\rangle + \beta|111\rangle$. How does creation of $|\Psi\rangle$ differ from cloning copies of $|\psi\rangle$?