Midterm

1. Consider a box with sides of length $L$ is in contact with a heat bath with temperature $T$. A collection of $N$ (non-interacting) particles all start out on the LHS of the the piston behind a partition. (16 points)

(a) A measurement of the particle positions is made prior to removing the partition and $K$ bits of information are retrieved. After the partition is removed the same measurement is made (after waiting for the system to equilibrate), how many bits of information are retrieved in this second measurement?

(b) How much heat will have been transferred to the system once thermal equilibrium is reached?

(c) What is the approximate minimal amount of time it will take for the system to reach thermal equilibrium once the partition is removed? Write your answer in terms of the temperature $T$, the mass $M$ of the particles and $L$. You need not worry about the overall factor. Hint: you will need to approximate the average velocity of the particles.

(d) What is the NET information loss of the total system?

(e) Suppose we put a GPS on each particle. What would the entropy be in the final state? Does your answer square with the second law of thermodynamics, if so how?

(f) Now the partition is replaced by a piston such that the gas slowly pushes the piston until its flush against the other side. What is the change in entropy of the gas after the process is complete and how much, if any, heat is transferred during the process.

(g) What is the NET information of the total system after this (problem (f)) process?

(h) Is it possible that we have GAINED information about the gas in the box? If so how?

2. Alice is sending Bob an atom which can be in one of five states. (6 points)

(a) Suppose she wants to transmit ten different (equally likely) messages to Bob. What is the upper bound on their probability for success.

(b) What is the maximum number of messages Alice could send so as to be sure that Bob successfully receives the message. What states would Alice use to send the message?
3. Find the Schmidt decomposition of the state \( | \psi \rangle = \frac{1}{\sqrt{3}}(|++\rangle + |--\rangle + |+-\rangle). \)
Is this state a product state or entangled? (this problem takes more computation than the others). (10 points)

4. Consider a composite system \( | \psi^{ABC} \rangle \). The Hamiltonian governing its dynamics is given by (12 points)
\[ H = \kappa (\sigma^A_z \otimes \sigma^B_x) \otimes 1^C. \]

(a) Suppose the initial state is \( \rho = \frac{1}{2}(|++\rangle\langle++| + |+-\rangle\langle-+|). \)
Is this a pure or mixed state?

(b) Determine the reduced density matrix \( \rho_A \).

(c) The systems is allowed to evolve until a time \( t \), determine \( \rho(t) \).

(d) Are there any instances in time where the state is pure?

(e) As the system evolves time calculate the net change in entropy.

5. Alice possess a Qbit \( | \psi \rangle = \frac{1}{\sqrt{2}}(\alpha |+\rangle + \beta |-\rangle) \). Alice and Bob share two Qbits that are in a Bell state (12 points)
\[ | \Phi^{(AB)}_{-} \rangle = \frac{1}{\sqrt{2}}(|-\rangle-|+\rangle), \]
such that the total states is \( | \psi \rangle \otimes | \Phi^{(AB)}_{-} \rangle \).

(a) What is the probability that when Alice makes a measurement on her two Qbits (one and two starting from the left) that she finds them in the state
\[ | \Psi^{(12)}_{+} \rangle = \frac{1}{\sqrt{2}}(|-\rangle+|+\rangle). \]

(b) Assuming she does indeed find the state \( | \Psi^{(12)}_{+} \rangle \) what will the state of Bobs Qbit be?

(c) Suppose Alice wants Bob to receive the initial state she was given \( | \psi \rangle \), she can tell Bob to perform a unitary on his state to end up with her initial state. What unitary should she tell him to perform?

(d) Given that Alices original state magically showed up instantanesouly in Bobs lab as soon as he performs the unitary, does this process violate any known laws of physics? If not why not?

6. Book PROBLEM 7.3 on page