1. A system of two spin-1/2 particles, particle a and particle b, is in the state 
\[ |\psi\rangle = (1/\sqrt{2}) (|z_a+, z_b+\rangle + |z_a-, z_b-\rangle) \in \mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b. \] Express each of the following properties in the form of a projector \( P \) acting in \( \mathcal{H} \) and briefly explain why the state \( |\psi\rangle \) has the property or lacks it.

i) Particle a has spin + in the x direction or it has spin − in the x direction.

ii) Both particles have spin + in the x direction.
iii) Both particles have spin + or both have spin − in the x direction.
2. Consider a spin-1 particle with basis states \{\ket{+}, \ket{0}, \ket{-}\} and Hamiltonian.

\[
H = \frac{\hbar \omega}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.
\]

a) A solution to Schrödinger’s equation \(i \hbar \frac{d}{dt} \ket{\psi_t} = H \ket{\psi_t}\) takes the form

\[
\ket{\psi_t} = c_+(t) \ket{-} + c_0(t) \ket{0} + c_-(t) \ket{+}.
\]

Write down a set of coupled differential equations satisfied by the complex coefficients \(c_+(t)\), \(c_0(t)\), and \(c_-(t)\).

b) Assuming \(\langle \psi_t | \psi_t \rangle = 1\), find the probability in terms of the \(c_j(t)\) that if an appropriate measurement is carried out at time \(t\), the system will be found in the state \(\ket{\phi}\), for

\[
(i): \ket{\phi} = \ket{0}, \quad (ii): \ket{\phi} = (\ket{+} - i \ket{-})/\sqrt{2}.
\]

Do NOT solve for \(c_j(t)!\)
c) A formal solution to Schrödinger’s equation can be written in the form

$$|\psi_t\rangle = T(t)|\psi_0\rangle, \text{ with } T(t) = e^{-iHt/\hbar}.$$ 

Express $T(t)$ as a $3 \times 3$ matrix in the usual $\{|+, |0\rangle, |-\rangle\}$ basis. Hint: the three eigenvectors of $H$ are $|a,b\rangle = \left(1/2, \pm 1/\sqrt{2}, 1/2\right)$ and $|c\rangle = (1, 0, -1)/\sqrt{2}$. 