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33-755 Quantum I Midterm Exam #1 (PM) Friday, Sept 27, 2019

1. Two spin-1/2's (adapted from Cohen-Tannoudji #IV.6 and IV.7)

Consider the state $|\psi_0\rangle = \alpha|++\rangle + \beta|+-\rangle + \gamma|-+\rangle + \delta|--\rangle$, where $|\pm\rangle$ are the up and down states along the $\hat{z}$ axis for each spin a and b.

(a) What condition must the coefficients satisfy so that the ket-vector is normalized?

(b) Under what condition is $|\psi_0\rangle$ a product state? Briefly explain how you know.

(c) Express the property "$S_{bx} = +\hbar/2$" as a matrix in the basis $\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$, then use this matrix to find the corresponding probability.

(d) Recall (from this morning!) that

$$S_{bx} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

and use it to calculate the mean value $\langle S_{bx} \rangle$.

(e) The state evolves from $|\psi_0\rangle$ for a time t under the Hamiltonian $H = \omega_1 S_{az} + \omega_2 S_{az}$. Express $|\psi(t)\rangle$ in the usual $|\pm\pm\rangle$ basis.

(f) Letting $\alpha = \beta = \gamma = \delta = 1/2$, determine the mean value $\langle S_{bx}\rangle(t)$.