NAME: ______________________________________

33-755 Quantum I Midterm Exam #2 Monday, Oct. 28, 2019

1. 3 Box Paradox

Consider a toy model with three locations for a particle. Call these three orthogonal states $|A\rangle$, $|B\rangle$ and $|C\rangle$. Take the time evolution to be trivial, $T = I$; if the particle is in one of the boxes, it stays there. We consider histories starting in the symmetric initial state

$$|\psi_0\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle + |C\rangle)$$

at time t_0 and ending in the final state

$$|F\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle - |C\rangle),$$

or its complement, at time t_2. The family of histories $\mathcal{A}$ defined by

$$[\psi_0] \odot \begin{cases} [A] \odot [F] & (Y^{(1)}) \\ [A] \odot (I - [F]) & (Y^{(2)}) \\ (I - [A]) \odot [F] & (Y^{(3)}) \\ (I - [A]) \odot (I - [F]) & (Y^{(4)}) \end{cases}$$

together with

$$Z \equiv (I - [\psi_0]) \odot I \odot I.$$

is complete, compatible and consistent.

(a) Evaluate the chain kets $|Y^{(1)}\rangle, \ldots, |Y^{(4)}\rangle$ and the probabilities $\Pr\,(Y^{(1)}), \ldots, \Pr\,(Y^{(4)})$.

(b) Demonstrate that $|Y^{(1)}\rangle$ is consistent with $|Y^{(2)}\rangle$, $|Y^{(3)}\rangle$, and $|Y^{(4)}\rangle$.

(c) Calculate the probability that the particle was in location A at t_1 given that it started in $|\psi_0\rangle$ at t_0 and ended in $|F\rangle$ at t_2. *Hint:* in which history(ies) do the various events occur at their respective times? Be sure to derive your answer algebraically using joint and conditional probabilities.

(d) An alternate family $\mathcal{B}$ utilizes property $[B]$ in place of $[A]$ at time t_1. You would find the same probability for location B at t_1 using family $\mathcal{B}$ as you obtained in part (c) for the probability of location A at t_1 using family $\mathcal{A}$. Nonetheless, your answers (assuming you solved (c) correctly) are in conflict. Briefly explain how you can reconcile these results.

2. Matrix elements of X and P (adapted from Cohen-Tannoudji, # 2.8)

Consider the Hamiltonian of a particle in one dimension,

$$H = \frac{P^2}{2m} + V(X),$$

with eigenvectors and eigenvalues $H|\varphi_n\rangle = E_n|\varphi_n\rangle$. Demonstrate that matrix elements of X and P are proportional, *i.e.*

$$\langle\varphi_n|P|\varphi_{n'}\rangle = \alpha_{n,n'}\langle\varphi_n|X|\varphi_{n'}\rangle,$$

and find the coefficient of proportionality $\alpha_{n,n'}$. *Hint:* the commutator $[X, P^2] = 2i\hbar P$ may be helpful.