Measurement

Beam splitter with detector

\[ H = H_p \otimes H_d \leftarrow \text{detector: basis \{1mz, 1n\}} \]

\[ T = S'R \]

\[ R = I_P \otimes I_d \text{ except: } R \mid 2c, n \rangle = \mid 2c, (1-n) \rangle \]

\[ S \mid mz, n \rangle = \mid m+1 \rangle \mid z, n \rangle \]

\[ S \mid 0a, n \rangle = \frac{1}{\sqrt{2}} (\mid 1a, n \rangle + \mid 1d, n \rangle) \]

Unitary evolution:

\[ \mid \psi_0 \rangle = \mid 0a, o \rangle \]

\[ \mid \psi_1 \rangle = \frac{1}{\sqrt{2}} (\mid 1c, o \rangle + \mid 1d, o \rangle) \]

\[ \mid \psi_2 \rangle = \frac{1}{\sqrt{2}} (\mid 2c, o \rangle + \mid 2d, o \rangle) \]

\[ \mid \psi_3 \rangle = \frac{1}{\sqrt{2}} (\mid 3c, o \rangle + \mid 3d, o \rangle) \]

Consistent history family:

Basis set \{ [mz, n] \} at each time

Histories with nonzero chain kets

\[ Y^c = [\psi_1] \otimes [1c, o] \otimes [2c, o] \otimes [3c, 1] \quad \mid Y^c \rangle = \frac{1}{\sqrt{2}} \mid 3c, 1 \rangle \quad P_r = \frac{1}{2} \]

\[ Y^d = [\psi_1] \otimes [1d, o] \otimes [2d, o] \otimes [3d, 1] \quad \mid Y^d \rangle = \frac{1}{\sqrt{2}} \mid 3d, 0 \rangle \quad P_r = \frac{1}{2} \]

Others vanish, e.g. \[ P_r([\psi_1] \otimes \cdots \otimes [3d, 1]) = 0 \]

\[ P_r(1c, 1 | 2c, 2) = P_r(Y^c) \div P_r(1c, 1) = \frac{1}{4} \div \frac{1}{4} = 1 \]

\[ P_r(1c, 1 | 1c, 1) = P_r(Y^c) \div P_r(1c, 1) = \frac{1}{4} \div \frac{1}{4} = 1 \]
Unitary history family: \[ |\psi_0\rangle \otimes [\Psi] \otimes [\Phi] \cdots \]
\[ |\psi_\nu\rangle \otimes (I_3 - a_1) \otimes [\Phi_\nu] \cdots \]

Fine family but: \[ [\psi_\nu][t z, n \hat{e}] = \frac{1}{\sqrt{\lambda}} |\psi_\nu\rangle \langle t z, n \hat{e}| \]
\[ [t z, n \hat{e}][\psi_\nu] = \frac{1}{\sqrt{\lambda}} |t z, n \hat{e}\rangle \langle \psi_\nu| \] \neq \text{ for } t \geq 1

Measurement of spin-1/2

Stern-Gerlach experiment:

Unitary evolution:
\[ |I_+\rangle, w \rightarrow |I_+\rangle, w' \rightarrow |I_+\rangle, w^+ \rightarrow \text{ flash } + \}
\[ |I_-\rangle, w \rightarrow |I_-\rangle, w' \rightarrow |I_-\rangle, w^- \rightarrow \text{ flash } - \}

destructive nondestructive

What if \[ |\psi\rangle = |x, w\rangle = \frac{1}{\sqrt{\lambda}} (|I_+\rangle, w + |I_-\rangle, w) \]?

Unitary history \[ [\psi_\nu] \otimes [\Psi] \otimes [\Phi] \] \[ \} \text{ incompatible because} \]

Measurement outcomes \[ \{ I_3 \otimes [w^+], I_3 \otimes [w^-]\} \]

\[ [\psi_\nu][w^+] \neq [w^+] [\psi_\nu] \]

Alternate family \[ [\psi_\nu] \otimes [\psi] \otimes \{[Z^+, w^+], [Z^-, w^-]\} \]

Last try \[ [\psi_\nu] \otimes \{[Z^+, w'], [Z^-, w']\} \leftrightarrow \gamma^+ \]

\[ [Z^-, w'] \otimes [Z^-, w'] \leftrightarrow \gamma^- \]
Pr([Z+1] | [W^*]) = Pr([Z+1] | [W^*]) = \frac{Pr([W^*])}{Pr(Y)} = Pr(Y) = 1 \checkmark

\therefore \text{ true measurement}