Kronig-Penney model

The figure below illustrates the potential of the Kronig-Penney model, which is a model for electrons in crystalline solids. The square wells have width $a$ and height $V_0$, and are placed periodically at intervals $L$, extending to $\pm \infty$. An electron of mass $m$ and energy $E$ is present.

(a) Determine the matrix $M$ that relates the coefficients of plane waves $e^{\pm ikx}$ in one valley (region where $V(x) = 0$) to the next.

(b) What condition must $M$ satisfy in order for a propagating solution to exist?

(c) Sketch the function $E(q)$ up to and slightly beyond the first band gap, taking $a = 2$, $L = 4$, $V_0 = \pi^2/4$, and setting $\hbar^2/2m = 1$. Here $q$ is the wavenumber of the Bloch function that obeys $\varphi_q(L + x)/\varphi_q(x) = e^{iqL}$. You may work out special points graphically and by hand and fill in the remainder qualitatively. Alternatively, you may use a code such as Mathematica or Python to solve a transcendental equation numerically.

(d) Notice the discontinuities in your function that correspond to “forbidden energies”. What would happen if an electron with forbidden energy were incident on a crystal with this potential energy function?