33-755 Sample History Problems, 2020

1. Consider the histories

$$X = [z+] \odot [x+] \odot [z-], \quad Y = [z+] \odot [x-] \odot [z-]$$

defined at times t_0, t_1 and t_2.

(a) Show that they are compatible.

(b) Assume trivial time evolution $T = I$. Are the histories consistent?

(c) Now let the spin evolve in the presence of a magnetic field along the y-axis. Assume the time evolution (expressed in the z-basis) is

$$T(t) = \begin{pmatrix} \cos \omega t/2 & -\sin \omega t/2 \\ \sin \omega t/2 & \cos \omega t/2 \end{pmatrix}$$

and define the three times as $t_0 = 0$, $t_1 = \pi/2\omega$ and $t_2 = \pi/\omega$. Are the histories X and Y consistent? Why or why not?

(d) Assuming initial condition $|z+\rangle$, determine the probabilities $\Pr(X)$ and $\Pr(Y)$.

2. Consider a random walk $\{s_j\}$ on the integers obeying $|s_{j+1} - s_j| \leq 1$ in which every history receives equal weight (i.e. $p = q = r = 1/3$). Suppose the initial probability distribution for s_0 is given by $\Pr(s_0 = 0) = 1/4$ and $\Pr(s_0 = 1) = 3/4$. Find the conditional probability $\Pr(s_1 = 1|s_3 = 2)$.

3. The figure (following page) shows a toy model with shift operator

$$S|0a\rangle = \frac{1}{\sqrt{2}}(|1a\rangle + |1b\rangle), \quad S|1a\rangle = \frac{1}{\sqrt{2}}(|2a\rangle + |2b\rangle),$$

$$S|0b\rangle = \frac{1}{\sqrt{2}}(-|1a\rangle + |1b\rangle), \quad S|1b\rangle = \frac{1}{\sqrt{2}}(-|2a\rangle + |2b\rangle).$$

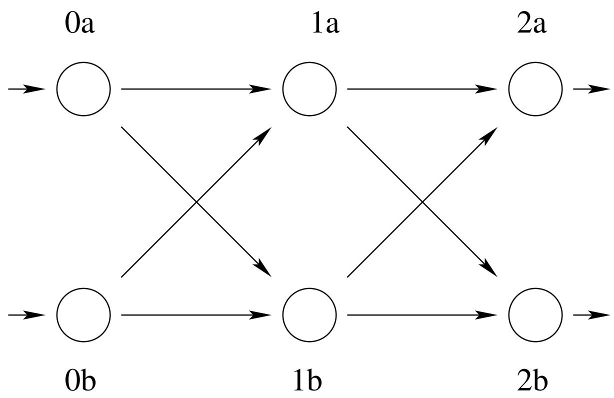
The histories of interest are from a family of the form

$$[\psi_0] \odot \{[1a], [1b], P\} \odot \{[2a], [2b], Q\}$$

at times $t = 0, 1, 2$, where $P = I - [1a] - [1b]$, $Q = I - [2a] - [2b]$.

(a) With time development operator T = S, show that if $[\psi_0] = [0a]$ this family is inconsistent.

(b) Find a $|\psi_0\rangle$ that is a superposition of $|0a\rangle$ and $|0b\rangle$, for which the family given above is consistent.



c) Now include a two-state measuring device $|n\rangle, n = 0$ or 1, and let $T = SR$, where $R = I \otimes I$ except for $R|1a, n\rangle = |1a, 1 - n\rangle$. Indicate on the toy model figure where this detector is located. Look at histories of the form given above except that $[\psi_0]$ is replaced by $]\Psi_0]$, where $|\Psi_0\rangle = |\psi_0\rangle \otimes |n = 0\rangle$. What can you say about the consistency of families for different choices of the initial particle state $|\psi_0\rangle$? Again consider only cases in which $|\psi_0\rangle$ is a superposition of $|0a\rangle$ and $|0b\rangle$.