1. Consider the histories

\[ X = [z+] \odot [x+] \odot [z-], \quad Y = [z+] \odot [x-] \odot [z-] \]

defined at times \( t_0, t_1 \) and \( t_2 \).

(a) Show that they are compatible.

(b) Assume trivial time evolution \( T = I \). Are the histories consistent?

(c) Now let the spin evolve in the presence of a magnetic field along the \( y \)-axis. Assume the time evolution (expressed in the \( z \)-basis)

\[
T(t) = \begin{pmatrix} \cos \omega t/2 & -\sin \omega t/2 \\ \sin \omega t/2 & \cos \omega t/2 \end{pmatrix}
\]

and define the three times as \( t_0 = 0, t_1 = \pi/2 \omega \) and \( t_2 = \pi/\omega \). Are the histories \( X \) and \( Y \) consistent? Why or why not?

(d) Assuming initial condition \( |z+\rangle \), determine the probabilities \( \Pr(X) \) and \( \Pr(Y) \).

2. Consider a random walk \( \{s_j\} \) on the integers obeying \(|s_{j+1} - s_j| \leq 1\) in which every history receives equal weight (i.e. \( p = q = r = 1/3 \)). Suppose the initial probability distribution for \( s_0 \) is given by \( \Pr(s_0 = 0) = 1/4 \) and \( \Pr(s_0 = 1) = 3/4 \). Find the conditional probability \( \Pr(s_1 = 1|s_3 = 2) \).

3. The figure (following page) shows a toy model with shift operator

\[
S|0a\rangle = \frac{1}{\sqrt{2}}(|1a\rangle + |1b\rangle), \quad S|1a\rangle = \frac{1}{\sqrt{2}}(|2a\rangle + |2b\rangle), \\
S|0b\rangle = \frac{1}{\sqrt{2}}(-|1a\rangle + |1b\rangle), \quad S|1b\rangle = \frac{1}{\sqrt{2}}(-|2a\rangle + |2b\rangle).
\]

The histories of interest are from a family of the form

\[
[\psi_0] \odot \{[1a], [1b], P\} \odot \{[2a], [2b], Q\}
\]

at times \( t = 0, 1, 2 \), where \( P = I - [1a] - [1b] \), \( Q = I - [2a] - [2b] \).

(a) With time development operator \( T = S \), show that if \( |\psi_0\rangle = |0a\rangle \) this family is inconsistent.

(b) Find a \( |\psi_0\rangle \) that is a superposition of \( |0a\rangle \) and \( |0b\rangle \), for which the family given above is consistent.
c) Now include a two-state measuring device $|n\rangle$, $n = 0$ or 1, and let $T = SR$, where $R = I \otimes I$ except for $R|1a, n\rangle = |1a, 1 - n\rangle$. Indicate on the toy model figure where this detector is located.

Look at histories of the form given above except that $|\psi_0\rangle$ is replaced by $|\Psi_0\rangle$, where $|\Psi_0\rangle = |\psi_0\rangle \otimes |n = 0\rangle$. What can you say about the consistency of families for different choices of the initial particle state $|\psi_0\rangle$? Again consider only cases in which $|\psi_0\rangle$ is a superposition of $|0a\rangle$ and $|0b\rangle$. 