NAME: _____

33-756 Midterm Exam 2 Monday, April 7, 2014

These three questions are adapted from Le Bellac. Most parts are easy and require almost no calculation.

1. The Ω^- hyperon is a ground state, spin $S = 3/2$ particle, composed of three strange quarks. Owing to the attractive strong interaction, the ground state wavefunction is nonzero at contact.

(a) Quarks are spin $S = 1/2$ particles. Express the spin state $|\chi\rangle = |j = 3/2, m_j = +3/2\rangle$ in the tensor product space of the three $|j = 1/2, m_j = \pm 1/2\rangle$ states. Is this spin state symmetrical or antisymmetrical under interchange of a pair of particles?

(b) Now consider the spatial state $\varphi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$. In the ground state, is this function symmetrical or antisymmetrical? Explain briefly.

(c) What can you conclude about the quarks that comprise the ground state Ω^- hyperon?

2. Low energy π^- mesons (spin 0) bind to a deuterium nucleus $^2\mathrm{H}^+$ in a hydrogenic $1s$ orbital state. This state undergoes a parity conserving nuclear decay to a pair of neutrons

$$\pi^- \quad + \quad {}^2\mathrm{H}^+ \quad \rightarrow \quad n \quad + \quad n.$$

(a) The deuterium nucleus consists of a proton (spin-1/2) plus a neutron (spin-1/2) in a $^3S_1$ state (recall spectroscopic notation $^{(2S+1)}L_J$). What is the total angular momentum j of the initial bound state (meson plus nucleus)?

(b) List all four possible final states consistent with the conservation of angular momentum but without considering the requirement of exchange antisymmetry. Your list should be given in spectroscopic notation.

(c) Explain why the final state must be ${}^3P_1$. Given that this is the case, what can be said about the parity of the π^- meson?

3. Consider a pair of identical spin-1/2 fermions in a potential well. The first excited state is $4x$ degenerate. Let the particles interact via the potential $V(\mathbf{r}_1, \mathbf{r}_2) = W\delta(\mathbf{r}_1 - \mathbf{r}_2)$, with $W > 0$. Into what spin states does the degenerate state split, what are the degeneracies of these states and how are they ordered in energy?