Jonah Waissman

PhD: Harvard/Weizmann (2014)

Shahal Ilani   Carbon nanotubes
scanning probe assembly

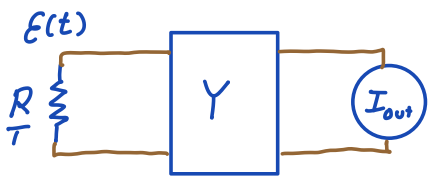
PostDoc: Harvard · Phil Kim
Electronic noise nanothermometer

Johnson - Nyquist noise (1928)

Johnson @ Bell Labs ← Transistor, Info theory, Unix, 9 Nobels 5 Turings

Nyquist @ AT&T ← Telephone monopoly 1885 – 1982 "Ma Bell"

Johnson



$$\overline{I_{out}^2} \sim R T |Y|^2$$

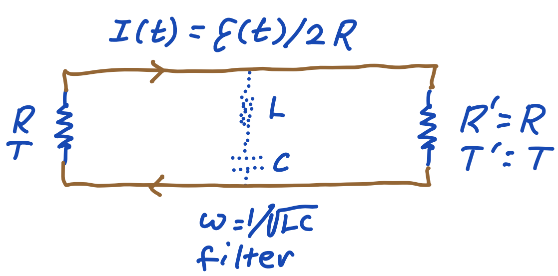
$$Y \doteq I_{out} / \mathcal{E}_{in}$$

Nyquist

$$\overline{I_{out}^2} = \frac{2}{\pi} k_B T \int_0^\infty R(\omega) |Y(\omega)|^2 d\omega$$

Universality

no assumptions on materials



$$\overline{P}_{R \to R'} = \overline{P'}_{R' \to R}$$

2^{nd} law

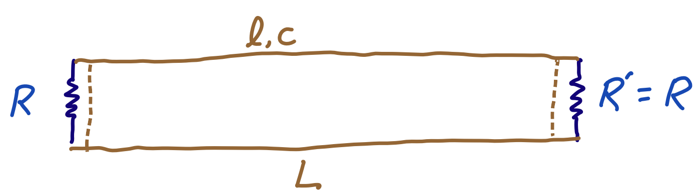
also: $P(\omega) = P'(\omega) \;\forall \omega$ proof: insert ω filter. If $P(\omega) > P'(\omega)$ then energy flux $R \to R'$ at equal T

$P(\omega) = ?$

Transmission line

inductance / length ℓ

capacitance / length c



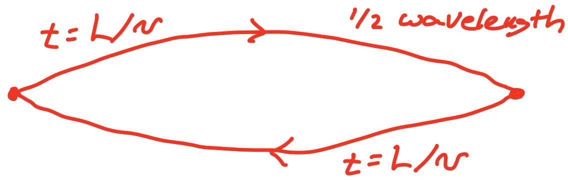
impedance $\sqrt{\ell / c}$ ← match impedance $= R \Rightarrow$ can short-circuit without reflection

Short-circuited transmission line

Total internal reflection

Nodes at ends

propagation time $2L/v$



frequency $\nu_n = \frac{v}{2L}\cdot n \qquad \omega_n = \frac{\pi v}{L}\cdot n$

Density of states: $D(\omega)\,d\omega = \frac{L}{\pi v}\,d\omega$

Equipartition

energy $k_B T/\text{mode} \qquad E(\omega)\,d\omega = k_B T \frac{L}{\pi v}\,d\omega$

power $\frac{1}{2}$ energy/time $\qquad P_{R\to R'} = \frac{k_B T}{2\pi}\,d\omega$

This power of 2 was missing.

$I = \mathcal{E}/2R \qquad P = I^2 R \Rightarrow \mathcal{E}^2 = 4PR = \frac{2}{\pi} k_B T R$

$$\overline{I^2_{out}} = \frac{2}{\pi} k_B T \int_0^\infty R(\omega)\,|Y(\omega)|^2\,d\omega$$

Caveat: Equipartition is false $\qquad E(\omega) = \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1} \sim k_B T$ for $\hbar\omega \ll k_B T$

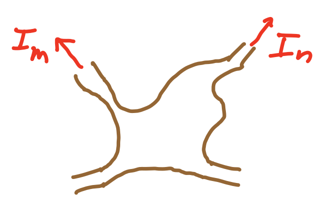
"Rayleigh-Jeans limit"

Multiterminal generalization (Sukhorukov + Loss, 1999)

$$S_{nm} \equiv \int_{-\infty}^{\infty} dt\,\langle \delta I_n(t)\,\delta I_m(0)\rangle = \int d\vec{r}\;\vec{\nabla}\phi_n \cdot \left(\overleftrightarrow{\sigma}\cdot\vec{\nabla}\phi_m\right)\Pi(\vec{r})$$

$$\Pi(\vec{r}) \equiv 2\int dE\, f(E,\vec{r})\,\left(1 - f(E,\vec{r})\right) \qquad f(E,\vec{r}) = \left(e^{(E - V(\vec{r}))/k_B T} + 1\right)^{-1}$$

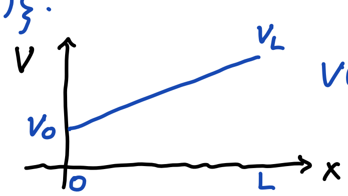
$$= 2 k_B T_e(\vec{r}) \quad \text{"hot electron temperature"}$$



Basis for potential $\{\phi_n(\vec{r})\}$:

$V(r) = \sum_n V_n \phi_n(\vec{r})$

$\sum_n \phi_n(\vec{r}) = 1$



$V(x) = V_0 + \frac{x}{L}(V_L - V_0)$

$\phi_0 = 1 - \frac{x}{L} \qquad \phi_L = \frac{x}{L}$

Derivation

1. Vector calculus $\vec{j}(\vec{r}) = -\overleftrightarrow{\sigma} \cdot \vec{\nabla} V \qquad \vec{\nabla}\cdot\vec{j} = G$

   Divergence theorem $\int_S \vec{E}\cdot d\hat{n} = \int_V \vec{\nabla}\cdot\vec{E}\, d\vec{r}$

2. Boltzmann equation in diffusive limit

   Distribution function $f(\vec{p},\vec{r},t)$

   $$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}_r + e\vec{E}\cdot\vec{\nabla}_p\right) f(\vec{p},\vec{r},t) - I[f] = \delta f$$

   Collision integrals ($I[f]$) — stochastic source (δf)

Related Concepts

1. Fluctuation-dissipation theorem:

Kubo (1966) dissipative forces $\longleftrightarrow$ fluctuations in conjugate variable

e.g. Brownian motion (Einstein, 1905) diffusion constant $D \equiv \lim_{t\to\infty} \frac{\langle x^2(t)\rangle}{2t}$

$D = \frac{k_B T}{\eta}$ viscosity $\eta \equiv -F/v$

Nyquist (1928) $\langle I_{in}^2 \rangle \sim k_B T/R$

2. Onsager reciprocal relations (1931):

Decay of spontaneous fluctuation $\equiv$ flow from region to reservoir

$\Rightarrow$ relations among kinetic coefficients

e.g. thermo-electric effects

particle flux $-J_N = \underbrace{L_{11}}_{T\sigma/e^2} \nabla \frac{\mu}{T} + \underbrace{L_{12}}_{-T^2\sigma Q/e} \nabla \frac{1}{T}$ — Thermopower; T gradient drives current "Seebeck effect"

heat flux $J_U = L_{21} \nabla \frac{\mu}{T} + L_{22} \nabla \frac{1}{T}$ — thermal conductivity without current "Fourier law"

$L_{21} = L_{12}$ $L_{22} = T^3\sigma Q^2 + T^2 K$

"Peltier effect" current carries heat $\boxed{\Pi = QT}$