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- PhD Harvard (2014) with Levitov (MIT): photoexcited hot electrons in graphene
- Postdoc CalTech with Rudner: predicted chiral Berry plasmons
- Singapore National Service (2016) Asst. Prof. NTU

AC conductivity (Drude model)

$$\dot{\vec{p}} = -\vec{p}/\tau - e\vec{E}$$

Let $p = p(\omega)e^{-i\omega t}$, $E = E(\omega)\,e^{-i\omega t}$, $\vec{j} = -ne\vec{p}/m$

$$\Rightarrow \quad -i\omega p(\omega) = -p(\omega)/\tau - eE(\omega)$$

$$j(\omega) = \underbrace{\left(\frac{ne^2\tau}{m}\right)\left(\frac{1}{1-i\omega\tau}\right)}_{\sigma(\omega)} E(\omega)$$

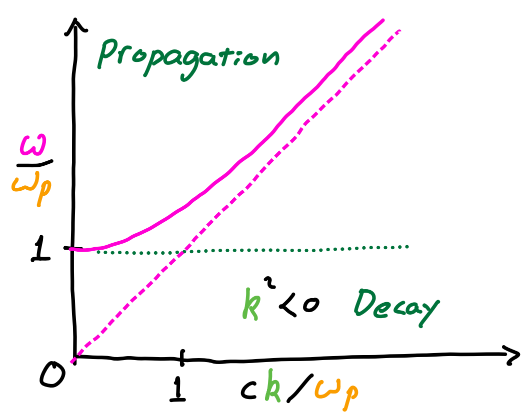
Wave propagation

$$\left.\begin{aligned} \vec{\nabla}\times\vec{E} &= -\frac{1}{c}\frac{\partial\vec{H}}{\partial t} \\ \vec{\nabla}\times\vec{H} &= \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t} \end{aligned}\right\} \Rightarrow -\nabla^2\vec{E} = \frac{\omega^2}{c^2}\left(1+\frac{4\pi i\sigma(\omega)}{\omega}\right)\vec{E}$$

$$\epsilon(\omega) \xrightarrow{\omega\tau \gg 1} 1 - \frac{\omega_p^2}{\omega^2} \qquad \omega_p^2 = \frac{4\pi ne^2}{m}$$

Try $E \sim e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$$\Rightarrow \quad k^2 = \epsilon(\omega)\frac{\omega^2}{c^2} \qquad \omega = \sqrt{c^2k^2+\omega_p^2}$$



$\omega < \omega_p$ k imaginary $\Rightarrow$ exponential decay

$\omega > \omega_p$ $k \in \mathbb{R}$ $\Rightarrow$ wave propagation

quantize as massive quasiparticle

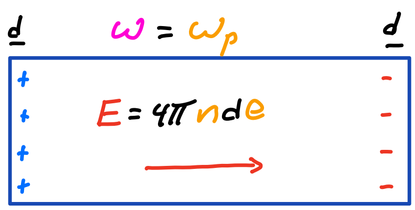
Transverse or Longitudinal?

EM wave in vacuum: $\vec{\nabla}\cdot\vec{E} = 4\pi\rho = 0 \Rightarrow \vec{k}\cdot\vec{E} = 0$ (transverse)

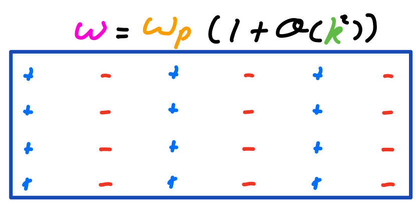
In matter, let $\rho(\vec{r}, t) = \rho(\omega)\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$, $\vec{j}(\omega) = \sigma(\omega)\vec{E}(\omega)$

Continuity $\vec{\nabla}\cdot\vec{j} = -\frac{\partial}{\partial t}\rho = i\omega\rho(\omega) \quad = \sigma(\omega)\, 4\pi\rho(\omega)$

Solvability $1 + \frac{4\pi i\sigma(\omega)}{\omega} = \epsilon(\omega) = 0 \Rightarrow \omega = \omega_L = \omega_p$

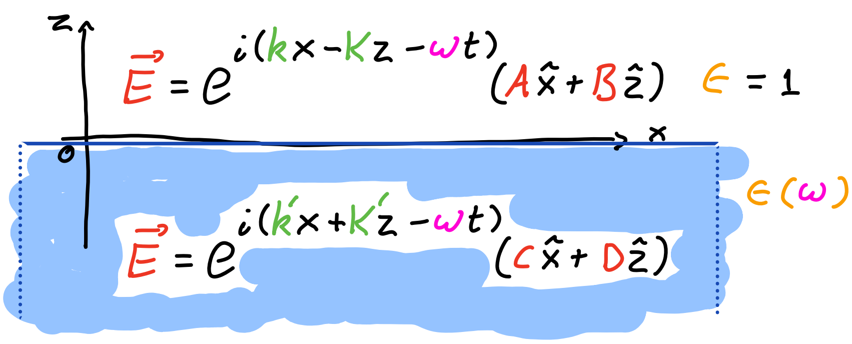


$k = 0$ plasmon



$k > 0$ plasmon

Surface plasmon



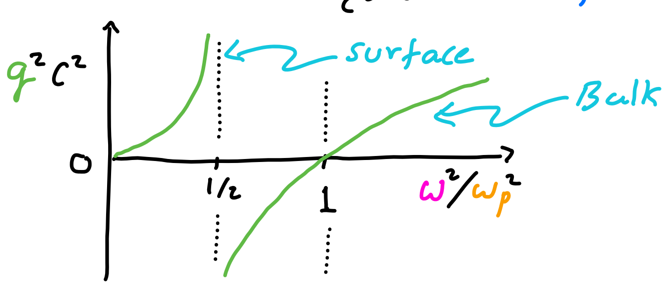
Boundary Conditions:

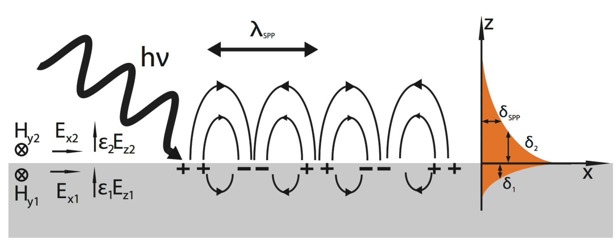
$[E_x] = [\epsilon E_z] = 0$

Gauss' Law: $\vec{\nabla}\cdot(\epsilon\vec{E}) = 0$

Wave equation: $-\nabla^2\vec{E} = \frac{\omega^2}{c^2}\epsilon\vec{E}$

Solve: $K'^2 - K^2 = \frac{\omega^2}{c^2}(1 - \epsilon(\omega))$, $K'/K = -\epsilon(\omega)$, $K^2 - q^2 = \frac{\omega^2}{c^2}$





Berry curvature (Xiao, Chang + Niu, 2010)

Recall adiabatic theorem: $H(t)$ slowly varying

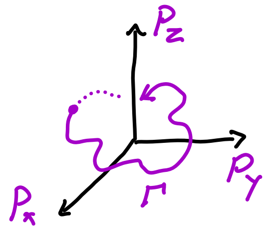
Solve in basis of instantaneous eigenstates $H(t)|n(t)\rangle = E_n(t)|n(t)\rangle$

$$|\psi_n(t=0)\rangle = |n(0)\rangle \Rightarrow |\psi_n(t)\rangle = e^{i\theta_n(t)} e^{i\gamma_n(t)} |n(t)\rangle$$

$$\theta_n(t) = -\frac{1}{\hbar}\int_0^t dt' E_n(t') \qquad \gamma_n(t) = i\int_0^t dt' \langle n(t')|\dot{n}(t')\rangle$$

dynamical phase Berry phase

Multidimensional generalization: $H = H(\underbrace{\vec{P}}_{\text{parameters}}(t))$



$$\gamma_n = \int_\Gamma d\vec{P}\cdot \underbrace{\langle n(\vec{P})|i\vec{\nabla}_{\vec{P}}\, n(\vec{P})\rangle}_{\vec{A}_n(\vec{P})}$$

Closed loop $\gamma = \oint_\Gamma d\vec{P}\cdot\langle n(\vec{P})|i\vec{\nabla}_{\vec{P}}\, n(\vec{P})\rangle = 2\pi p$ Gauge invariant

$$= \int_S d\vec{S}\cdot\vec{\Omega}_n(\vec{P}) \qquad \vec{\Omega}_n = \vec{\nabla}_{\vec{P}}\times\vec{A}_n$$

Band structure: $\psi_{n\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{n\vec{k}}(\vec{r}) \qquad H\psi_{n\vec{k}}(\vec{r}) = E_n(\vec{k})\psi_{n\vec{k}}(\vec{r})$

$$\vec{\Omega}_n(\vec{k}) = \vec{\nabla}_{\vec{k}}\times\langle u_{n\vec{k}}|i\vec{\nabla}_{\vec{k}} u_{n\vec{k}}\rangle$$

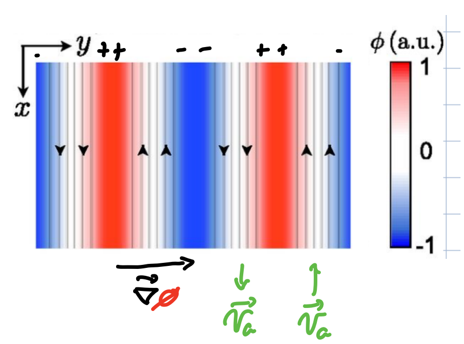
Anomalous velocity (semiclassical dynamics):

$$\hbar\dot{\vec{k}} = \dot{\vec{p}} = -e\vec{E} - \frac{e}{c}\vec{v}\times\vec{B}$$

$$\dot{\vec{r}} = \vec{v} = \frac{1}{\hbar}\frac{\partial E(\vec{k})}{\partial\vec{k}} - \dot{\vec{k}}\times\vec{\Omega}(\vec{k})$$

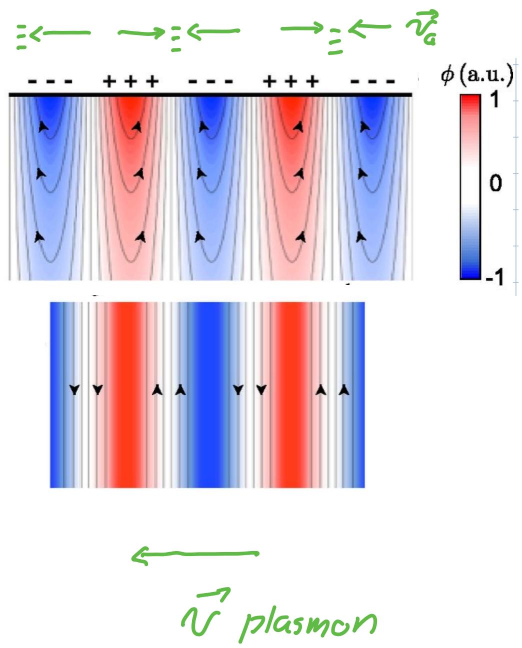
$$\vec{v}_a \equiv \frac{eF}{\hbar}\vec{\nabla}\phi\times\hat{z}$$

$$F = \sum_n\int\frac{d\vec{k}}{(2\pi\hbar)^2}\Omega_n^{(z)}(\vec{k})f_n(\vec{k})$$



Berry flux $\mathcal{F} \neq 0$ for broken time or space inversion symmetry

1. Anomalous Hall systems (e.g. magnetically doped TIs)
2. Nonmagnetic systems out of equilibrium (e.g. photo-excited gapped Dirac systems)
3. Surfaces and edges



Surface — Electrons forced to left of electron density ⇒ enhances leftward motion

Bulk — diminishes rightward

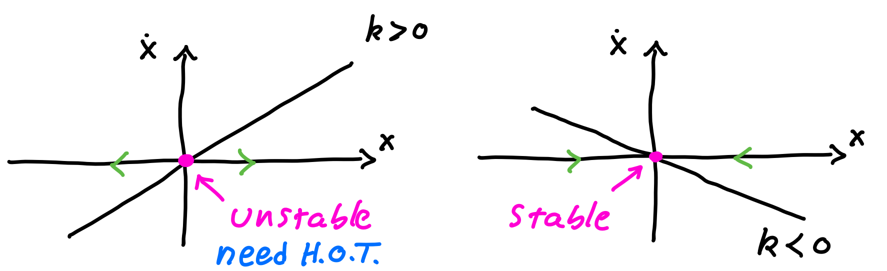
Bifurcations (Strogatz: Nonlinear dynamics and chaos)

Motion in 1-D: $m\ddot{x} + \lambda \dot{x} = f(x) = -\frac{\partial V}{\partial x}$ over-damping ⇒ drop $m\ddot{x}$

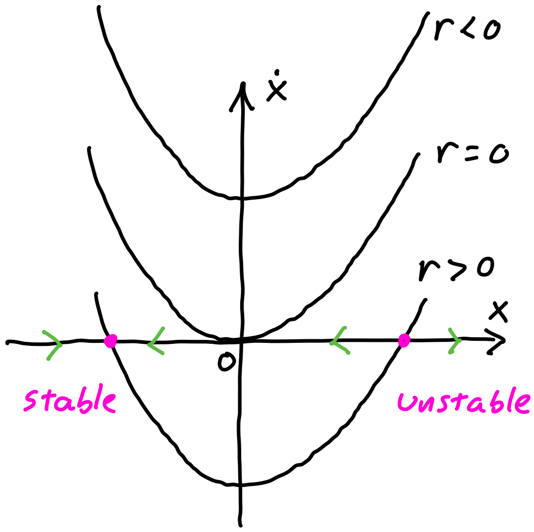
"Normal forms": minimal models, e.g. $f(x) = kx$

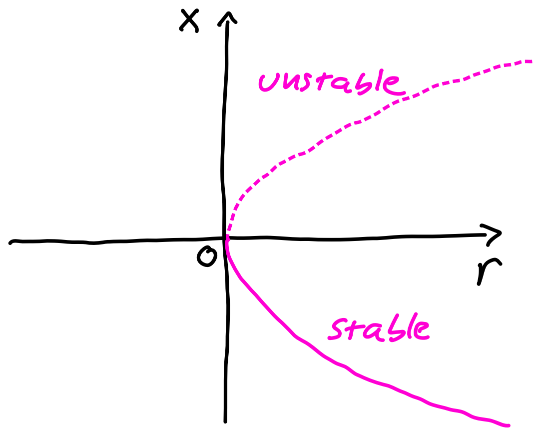
Example: $\dot{x} = kx$

fixed point $x = 0$



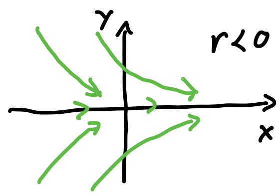
Saddle-Node: $\dot{x} = x^2 - r$ Fixed points $x = \pm\sqrt{r}$

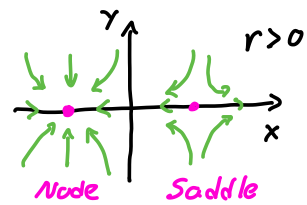




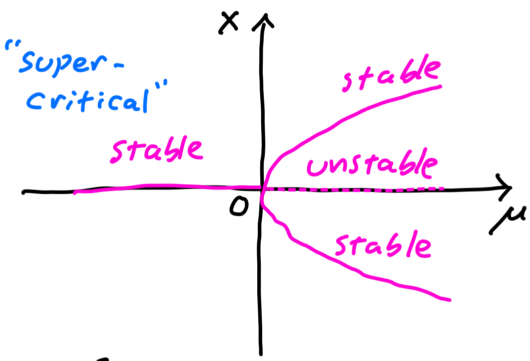
2D Saddle-Node: $\dot{x} = x^2 - r$

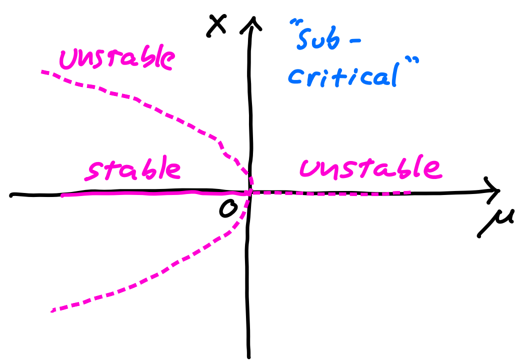
$\dot{y} = -y$





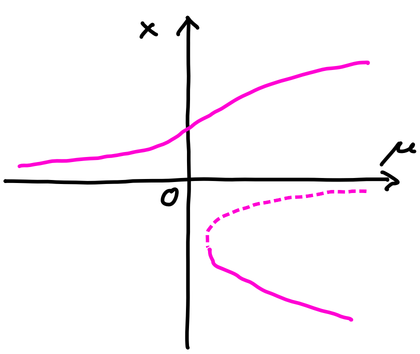
Pitchfork: $\dot{x} = \mu x - x^3$ $\dot{x} = \mu x + x^3$





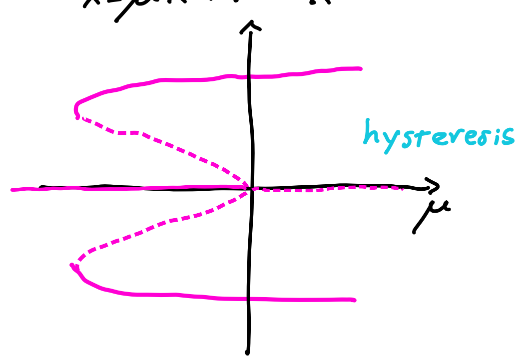
$\dot{x} = h + \mu x - x^3$

$h > 0$



Catastrophe

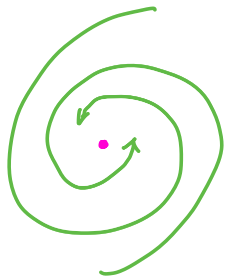
$\dot{x} = \mu x + x^3 - x^5$



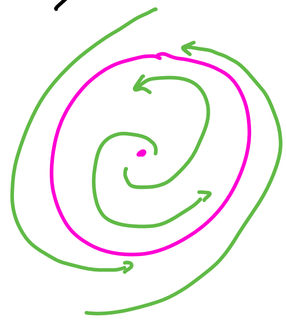
Hopf bifurcation:

$$\dot{r} = \mu r - r^3$$
$$\dot{\theta} = \omega + b r^2$$

$\mu < 0$



$\mu > 0$



Limit cycle