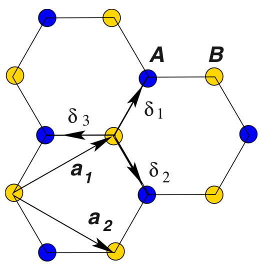
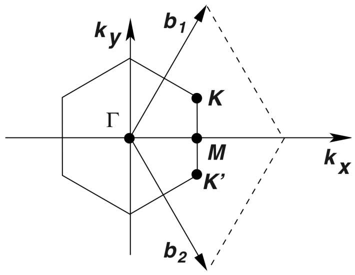
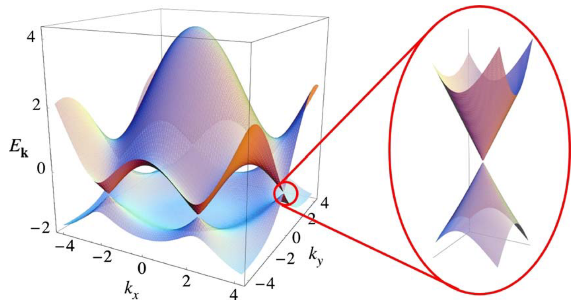
Graphene



Castro Neto 2009





$$H = -t \sum_{\langle ij \rangle s} \left(a^\dagger_{is} b_{js} + b^\dagger_{js} a_{is} \right)$$

$$E_\pm = \pm t \sqrt{3 + f(\vec{k})} \qquad f(\vec{k}) = 2\cos(\sqrt{3}k_y a) + 4\cos\left(\tfrac{\sqrt{3}}{2}k_y a\right)\cos\left(\tfrac{3}{2}k_x a\right)$$

Dirac point: $H_K = -i v_F \vec{\sigma} \cdot \vec{\nabla} \qquad H_{K'} = -i v_F \vec{\sigma}^* \cdot \vec{\nabla}$

Generalize: $H = \frac{\sqrt{3}}{2} a t \left(q_x \tau_z \sigma_x + q_y \sigma_y \right) + \frac{\Delta}{2} \sigma_z$

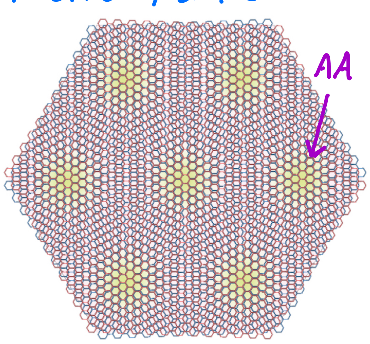
$\vec{q} = \vec{k} - \vec{K}^{(\prime)}$ valley index mass $\Delta = 0$

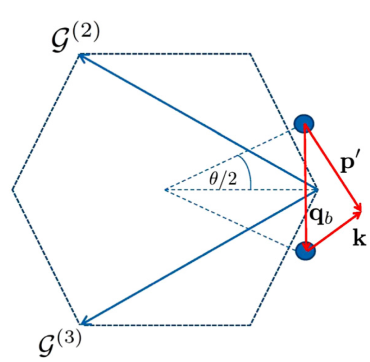
$\tau_z = \pm 1$ (chiral symmetry)

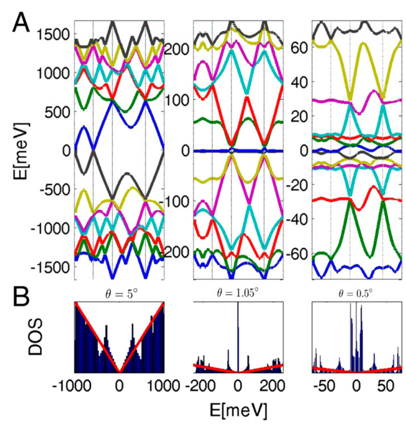
Twisted bilayers

Bistritzer & MacDonald (2011)

Moiré pattern







Annimation (and superconductivity!) Cao, ⋯, Jarillo-Herrero (2018)

$$H_0 = |1\rangle h(\theta/2)\langle 1| + |2\rangle h(-\theta/2)\langle 2|$$

layer   twist θ

single layer: $h(\theta) = -vk \begin{pmatrix} 0 & e^{i(\theta_k - \theta)} \\ e^{-i(\theta_k - \theta)} & 0 \end{pmatrix}$

angle of $\vec{k}$ relative to $\hat{x}$

Layer coupling matrix elements

$$T_{kp'}^{\alpha\beta}(\vec{r}) = w \sum_{j=1}^{3} e^{-i\vec{q}_j \cdot \vec{r}}\, T_j^{\alpha\beta}$$

α, β sublattice indices layer 1, 2

$|\vec{q}_j| = |\vec{k}|$

$$T_1 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad T_2 = \begin{pmatrix} e^{-2\pi i/3} & 1 \\ e^{2\pi i/3} & e^{-2\pi i/3} \end{pmatrix} \quad T_3 = T_2^*$$

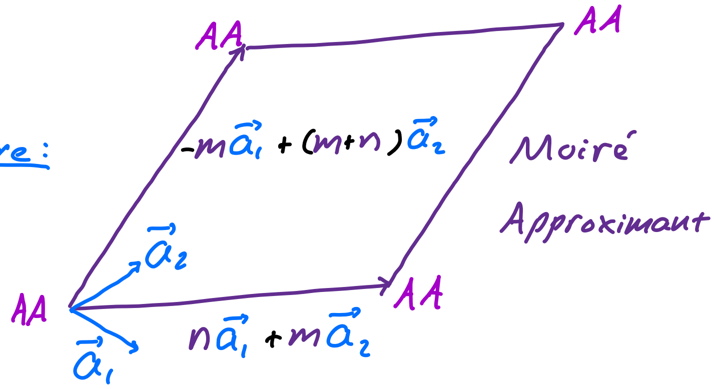
Magic angle origin

Trambly de Laissardière:

Confinement in AA regions

$$D_{AA} = m^2 + mn + n^2$$

$$\cos\theta = \frac{m^2 + 4mn + n^2}{2(m^2 + mn + n^2)}$$



$m = n+1 \Rightarrow \theta$ small

Resonance condition: $k D_{AA} = 2\pi p \quad p \in \mathbb{Z} \quad \theta_p \approx 1/p$

Grigory Tarnopolsky:

$$H = \begin{pmatrix} -iv\vec{\sigma}_{\theta/2} \cdot \vec{\nabla} & T(\vec{r}) \\ T^\dagger(\vec{r}) & -iv\vec{\sigma}_{-\theta/2} \cdot \vec{\nabla} \end{pmatrix}$$

Same as B.M.

Drop diagonal part of T (AA coupling) $\Rightarrow$ chiral symmetry

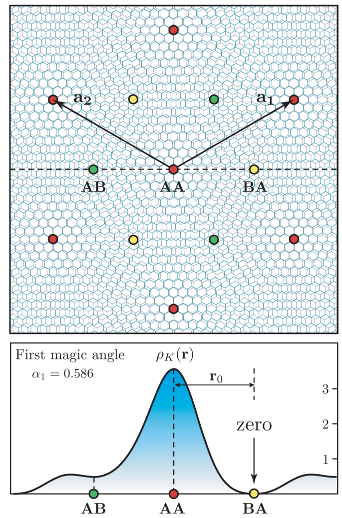
$E_\pm = \pm E_+$ $E = 0$ states appear in chiral pairs concentrated on A/B sublattices

Combine chiral and C_3 rotations:

$\exists\ \vec{k}$ s.t. $E_\pm(\vec{k}) = 0$ (i.e. bands touch)

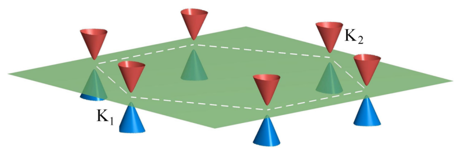
if $\psi_k(\vec{r}_{BA}) = 0$ then $E_\pm(\vec{k}) = 0 \quad \forall\ \vec{k}$

reintroducing $w_{AA} \Rightarrow$ small change in $E_\pm$



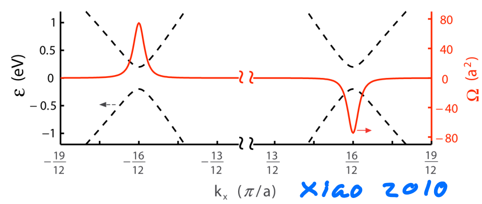
Chern numbers

Quantized integral of Berry curvature in gapped Dirac cones



$$C = \frac{1}{4\pi}\int dA\ \Omega$$

$C = +1$



Xiao 2010

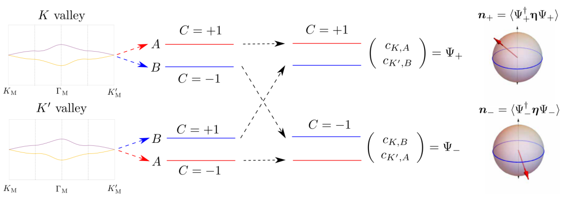
$\leftarrow C = -1$

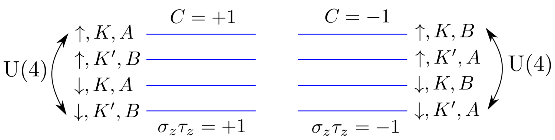
Magic angle hidden symmetry

Quantum numbers	Valley	K/K'	$\tau_z = \pm 1$ (pseudospin)
	spin	$\uparrow/\downarrow$	$s_z = \pm 1$
	Band	$+/-$	$b = \pm 1$

Approximately degenerate $\Rightarrow$ Change basis Band $\rightarrow$ Sublattice

$\sigma_z = A/B$





Term	Symmetry	Energy scale
U_S	$\mathrm{U}(4) \times \mathrm{U}(4)$	15-25 meV
t_S	$\mathrm{U}(4)_R$	4-6 meV
U_A	$\mathrm{U}(4)_{\mathcal{PT}}$	4-6 meV
t_A	$\mathrm{U}(2)_K \times \mathrm{U}(2)_{K'}$	0.5-1 meV

From valley/sublattice to Chern/pseudospin

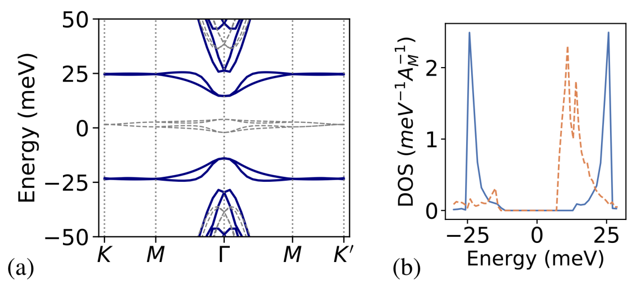
$(\boldsymbol{\tau}, \boldsymbol{\sigma}) \to (\boldsymbol{\gamma}, \boldsymbol{\eta})$

$\boldsymbol{\gamma} = (\gamma_x, \gamma_y, \gamma_z) = (\sigma_x, \sigma_y\tau_z, \sigma_z\tau_z)$

$\boldsymbol{\eta} = (\eta_x, \eta_y, \eta_z) = (\sigma_x\tau_x, \sigma_x\tau_y, \tau_z)$

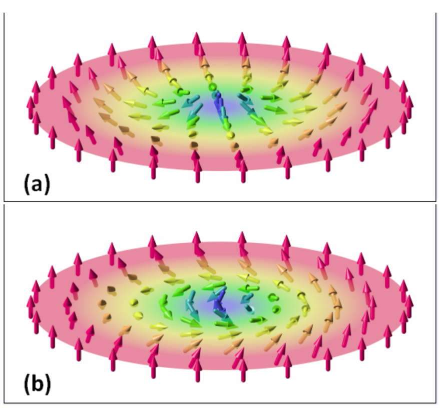
Basis		
Valley	$\tau_z = K/K'$	
Sublattice	$\sigma_z = A/B$	
Chern sector	$\gamma_z = \sigma_z\tau_z = +/-$	
Pseudospin	$\eta_z = \tau_z = \uparrow_{ps}/\downarrow_{ps}$	
Symmetries		
Symm	(τ, σ) basis	(γ, η) basis
$\mathcal{T}$	$\tau_x\mathcal{K}$	$\gamma_x\eta_x\mathcal{K}$
C_2	$\sigma_x\tau_x$	η_x
$U_V(1)$	$e^{i\phi\tau_z}$	$e^{i\phi\eta_z}$

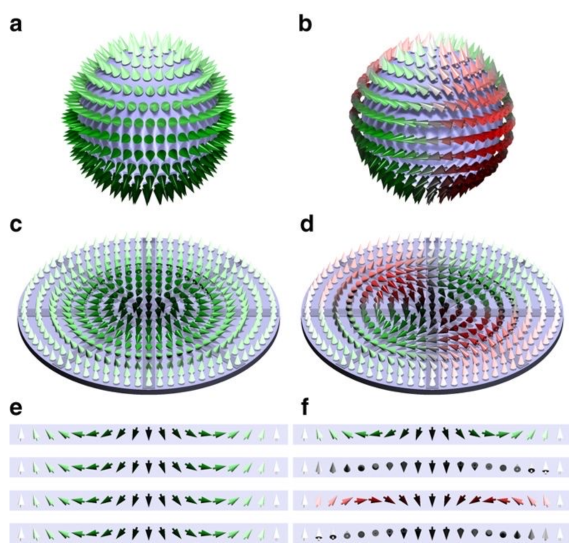
New symmetry $U(4) \times U(4)$



Magnetic Skyrmions

$$n = \frac{1}{4\pi} \int dA \, \vec{M} \cdot \left(\frac{\partial \vec{M}}{\partial x} \times \frac{\partial \vec{M}}{\partial y} \right)$$





Pseudospin Skyrmion

$$\vec{n}_{\pm} = \langle \Psi_{\pm} | \vec{\eta} | \Psi_{\pm} \rangle$$

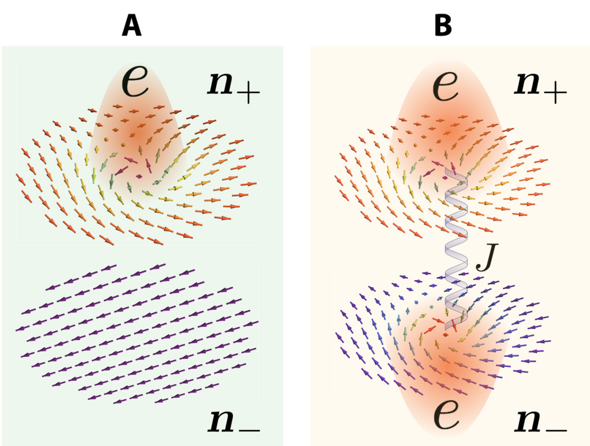
Skyrmion topological density $q_{\pm}(\vec{r}) = \frac{1}{4\pi} \vec{n}_{\pm} \cdot \left(\frac{\partial \vec{n}_{\pm}}{\partial x} \times \frac{\partial \vec{n}_{\pm}}{\partial y} \right)$

Electric charge density $\rho(\vec{r}) = e\,(q_{+}(\vec{r}) - q_{-}(\vec{r}))$

Estimated to be lowest energy charged excitations

Chern $C = \pm 1$ Skyrmions bind antiferromagnetically

Proposal: pairing of skyrmions leads to superconductivity



Estimated $T_c \sim 1\text{-}5$ K