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Article

Cascade of correlated electron states in the kagome superconductor CsV_3Sb_5

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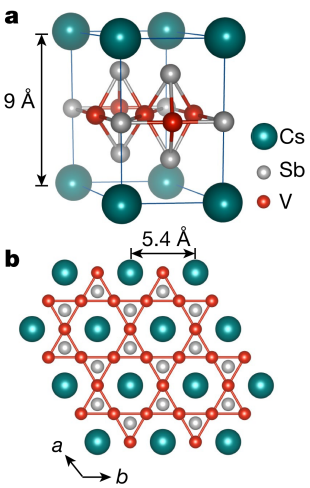
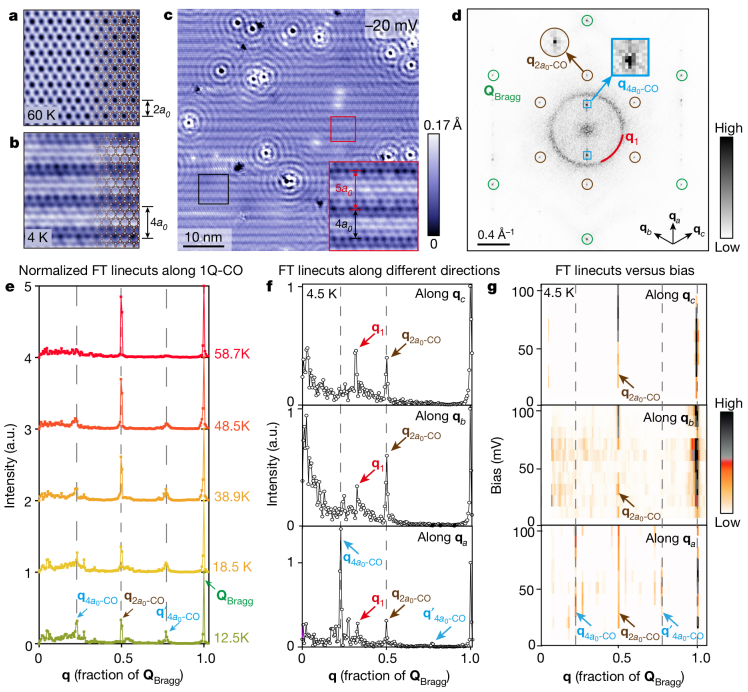


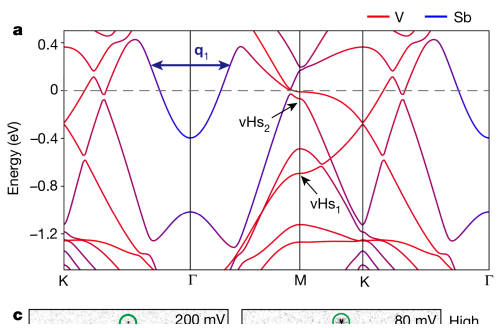
STM Topography

Fourier Transform



"Frustration"

Can lead to spin liquids, charge ordering, ...



Van Hove singularities

Saddle points at M

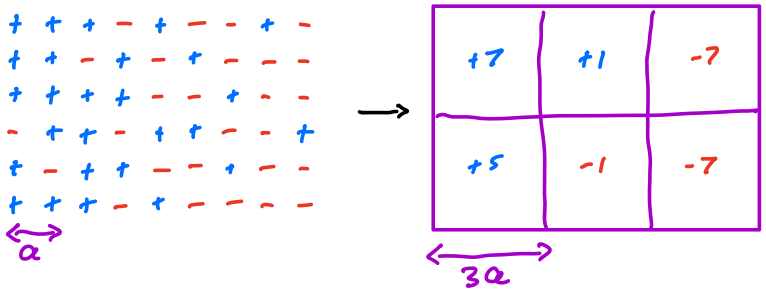
$E(\vec{q}) = aq_x^2 - bq_y^2$

Landau theory (ocw.mit.edu/ search 8-334)

Ising model: $H = -\frac{1}{2} K \sum_{\langle ij \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$

Free energy: $F = E - TS = -T \ln Z \qquad Z = \sum_{\{\sigma\}} e^{-H/T}$

Coarse grain: $\sigma_i \to m(\vec{r})$



Model free energy $f(T,h;m)$ ← m is dependent variable!

$$f(m) = \frac{1}{2}(T - T_c) m^2 + u m^4 + \cdots + \frac{1}{2} K |\vec{\nabla} m|^2 + \mathcal{O}((\nabla^2 m)^2) - hm$$

$S \approx \ln 2 - \frac{1}{2} m^2$

$T_c \sim K$ — K favors $m \neq 0$

$u > 0$ so m is finite

drop higher powers

$K > 0$ favors alignment

drop higher derivatives

h breaks symmetry

Vary $m(\vec{r})$ to minimize f: $\vec{\nabla} m = 0$, $\bar{m} = \pm\sqrt{(T_c - T)/4u}$ if $h = 0$

$\bar{m} = h/|T - T_c|$ if $h \neq 0$

Critical exponents $\beta = 1/2$ $\gamma = 1$

actual $1/8, 0.327, 1/2$ actual $7/4, 1.237, 1$ in dimension $2, 3, 4$

Fluctuations $m(\vec{r}) = \bar{m} + \phi(\vec{r})$ $T - T_c$

$$f(\phi) = f(\bar{m}) + \int \frac{d^d \vec{r}}{V} \left\{ \frac{K}{2} |\vec{\nabla} \phi(\vec{r})|^2 + \frac{1}{2} (t + 4u\bar{m}^2) \phi^2(\vec{r}) \right\}$$

Correlations $\langle \phi(\vec{r}_1) \phi(\vec{r}_2) \rangle = \int \mathcal{D}\phi(\vec{r}) \; \phi(\vec{r}_1) \phi(\vec{r}_2) \; e^{-f(\phi)/T}$

Evaluate Gaussian integrals in $\vec{k}$-space: $|\vec{k}| < \Lambda = \frac{\pi}{a}$

$\langle \phi(\vec{r}_1)\phi(\vec{r}_2)\rangle \sim e^{-|\vec{r}_1-\vec{r}_2|/\xi}$ $\quad \xi \sim \frac{1}{\sqrt{Kt}}$ diverges as $t \to 0$

Example: Heat capacity $C \sim \frac{1}{K^2}\begin{cases} a^{4-d} & d \geq 4 \\ \xi^{4-d} & d < 4 \end{cases}$

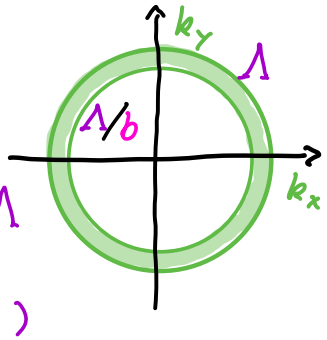
Renormalization Group (Still ocw.mit.edu 8-334)

1. Coarse grain $a \to ba$, $b > 1$, $\bar{m}(\vec{r}) = \frac{1}{b^d}\int_{b(\vec{r})} d^d\vec{r}\, m(\vec{r})$
2. Rescale $\vec{r} \to \vec{r}/b$
3. Renormalize: new $m'(\vec{r}') = \bar{m}(b\vec{r})/\zeta$ to match fluctuations
4. Repeat until converged

Example: Gaussian Model

Keep only terms up to $O(m^2)$

$$Z = \int \prod_{|\vec{k}|<\Lambda} dm(\vec{k})\, e^{-\beta f(t,K,L,h;\{m(\vec{k})\})}$$

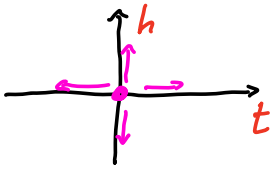


Coarse grain: eliminate fluctuations $\Lambda/b < |\vec{k}| < \Lambda$

$$\{m(\vec{k})\} = \{m_<(\vec{k}_<)\} \cup \{\sigma_>(\vec{k}_>)\} \Rightarrow f = f(m_<, \sigma_>)$$

$$Z = \int \prod_{k_<} dm_< \prod_{k_>} d\sigma_>\, e^{-\beta f(m_<,\sigma_>)} \equiv \int \prod_{k_<} dm_<\, e^{-\beta f'(m_<)}$$

Renormalized $f' = f(t', K', L', h')$



$t' = b^2 t$ and $h' = b^{1+d/2} h$ grow "relevant"

$K' = K$ is "marginal" $L' = L/b^2$ vanishes "irrelevant"

Power counting: m^n scales as b^s $\quad s = d + n(2-d)/2$

s	$n=1$	2	4	6
$d=4$	3	2	0	-2
3	$2\frac{1}{2}$	2	1	0
2	2	2	2	2
1	$1\frac{1}{2}$	2	3	4

$u m^4$ is marginal in $d=4$, relevant in $d<4 \Rightarrow$ expansion in $\epsilon = 4-d$! (Wilson-Fisher)

RG for quantum systems

1. Quantum in d dimensions = classical in $d+1$ dimensions

Suzuki-Trotter time slicing $\Rightarrow$ "near-neighbor interaction" in time

2. "Integrate out" high energy terms

e.g. $H = H_L + H_H + V + V^\dagger$: $\begin{pmatrix} H_L & V^\dagger \\ V & H_H \end{pmatrix} = U \begin{pmatrix} \tilde{H}_L & 0 \\ 0 & \tilde{H}_H \end{pmatrix} U^\dagger$

progressively reduce energy range above E_F.

Compare: $H = \begin{pmatrix} E_L & V \\ V & E_H \end{pmatrix}$ $\quad E_- \approx E_L + \dfrac{V^2}{E_H - E_L}$ $\quad E_+ \approx E_H - \dfrac{V^2}{E_H - E_L}$

↑ virtual fluctuations out of L and back

Editors' Suggestion

Electronic instabilities of kagome metals: Saddle points and Landau theory

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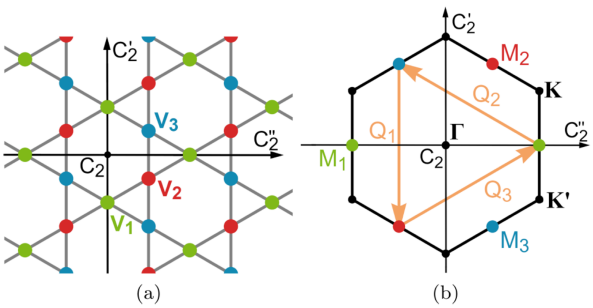
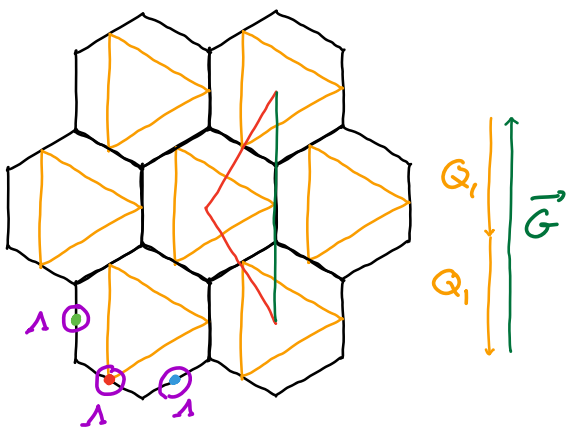
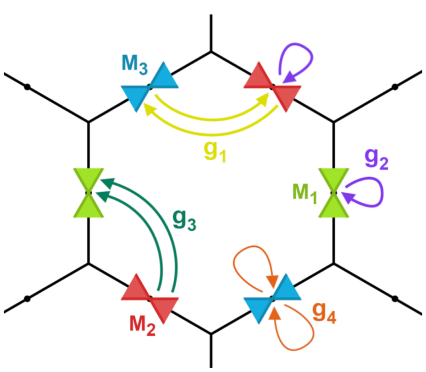


Vanadium lattice and B.Z.

Nesting $\{\vec{Q}_\alpha\}$ connects M points

$\vec{Q}_\alpha + \vec{G} = -\vec{Q}_\alpha$ "umklapp"





patches — M-points

$$H_1 = \frac{1}{2\mathcal{N}} \sum_{|\boldsymbol{q}_1|,\ldots,|\boldsymbol{q}_4|<\Lambda}{}' \Bigg[\sum_{\alpha\neq\beta} \Big(g_1 c^\dagger_{\alpha\boldsymbol{q}_1\sigma} c^\dagger_{\beta\boldsymbol{q}_2\sigma'} c_{\alpha\boldsymbol{q}_3\sigma'} c_{\beta\boldsymbol{q}_4\sigma} + g_2 c^\dagger_{\alpha\boldsymbol{q}_1\sigma} c^\dagger_{\beta\boldsymbol{q}_2\sigma'} c_{\beta\boldsymbol{q}_3\sigma'} c_{\alpha\boldsymbol{q}_4\sigma} + g_3 c^\dagger_{\alpha\boldsymbol{q}_1\sigma} c^\dagger_{\alpha\boldsymbol{q}_2\sigma'} c_{\beta\boldsymbol{q}_3\sigma'} c_{\beta\boldsymbol{q}_4\sigma} \Big) + \sum_\alpha g_4 c^\dagger_{\alpha\boldsymbol{q}_1\sigma} c^\dagger_{\alpha\boldsymbol{q}_2\sigma'} c_{\alpha\boldsymbol{q}_3\sigma'} c_{\alpha\boldsymbol{q}_4\sigma} \Bigg],$$

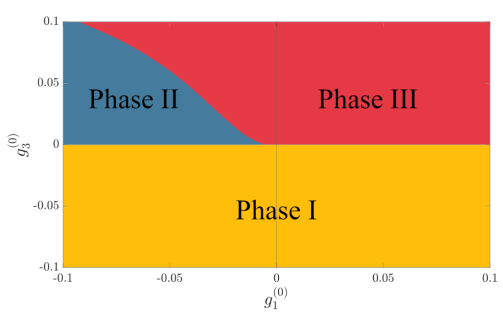
R.G. flow equations

$$\frac{dg_1}{dy} = 2d_1 g_1 (g_2 - g_1),$$

$$\frac{dg_2}{dy} = d_1 \left(g_2^2 + g_3^2\right),$$

$$\frac{dg_3}{dy} = -g_3^2 - 2g_3 g_4 + 2d_1 g_3 (2g_2 - g_1),$$

$$\frac{dg_4}{dy} = -2g_3^2 - g_4^2,$$



Landau theory order parameters

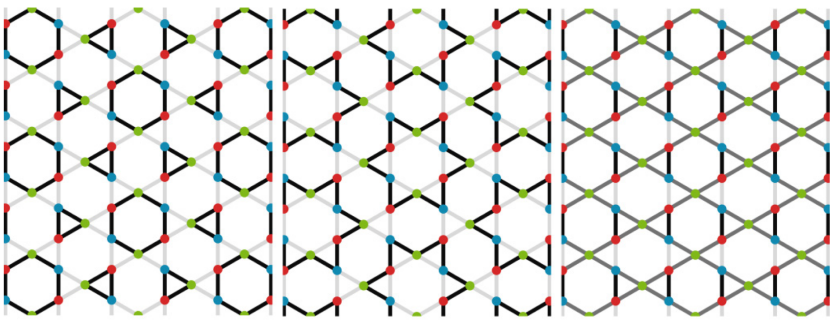
OP	Definition	Interaction strength (G)
rCDW	$N_\alpha = G_{\mathrm{rCDW}} \frac{\lvert\epsilon_{\alpha\beta\gamma}\rvert}{2\mathcal{N}} \sum_{\boldsymbol{q}} \langle c^\dagger_{\beta\boldsymbol{q}} c_{\gamma\boldsymbol{q}} \rangle$	$-2g_1 + g_2 - g_3$
iCDW	$\phi_\alpha = G_{\mathrm{iCDW}} \frac{\epsilon_{\alpha\beta\gamma}}{2i\mathcal{N}} \sum_{\boldsymbol{q}} \langle c^\dagger_{\beta\boldsymbol{q}} c_{\gamma\boldsymbol{q}} \rangle$	$-2g_1 + g_2 + g_3$
rSDW	$\boldsymbol{S}_\alpha = G_{\mathrm{rSDW}} \frac{\lvert\epsilon_{\alpha\beta\gamma}\rvert}{2\mathcal{N}} \sum_{\boldsymbol{q}} \langle c^\dagger_{\beta\boldsymbol{q}} \frac{\boldsymbol{\sigma}}{2} c_{\gamma\boldsymbol{q}} \rangle$	$g_2 + g_3$
iSDW	$\boldsymbol{\psi}_\alpha = G_{\mathrm{iSDW}} \frac{\epsilon_{\alpha\beta\gamma}}{2i\mathcal{N}} \sum_{\boldsymbol{q}} \langle c^\dagger_{\beta\boldsymbol{q}} \frac{\boldsymbol{\sigma}}{2} c_{\gamma\boldsymbol{q}} \rangle$	$g_2 - g_3$
sSC	$\Delta_s = G_{\mathrm{sSC}} \frac{1}{\sqrt{3}\mathcal{N}} \sum_{\boldsymbol{q}} \langle c_{\alpha\boldsymbol{q}\downarrow} c_{\alpha-\boldsymbol{q}\uparrow} \rangle$	$-2g_3 - g_4$
dSC	$\Delta_{xy} = G_{\mathrm{dSC}} \frac{1}{\mathcal{N}} \times \sum_{\boldsymbol{q}} \langle c_{\boldsymbol{q}\downarrow} D_{xy} c_{-\boldsymbol{q}\uparrow} \rangle$	$g_3 - g_4$
	$\Delta_{x^2-y^2} = G_{\mathrm{dSC}} \frac{1}{\mathcal{N}} \times \sum_{\boldsymbol{q}} \langle c_{\boldsymbol{q}\downarrow} D_{x^2-y^2} c_{-\boldsymbol{q}\uparrow} \rangle$	

$$F_{\mathrm{CDW}} = \sum_\alpha N_\alpha^2 + \sum_\alpha \phi_\alpha^2 - \mathrm{Tr} \ln G^{-1}$$

charge density

orbital moment

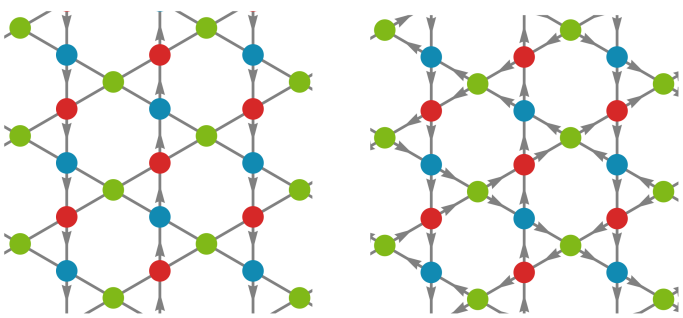
Charge density patterns



(a) (b) (c)

"3Q trCDW" "3Q -rCDW" "1Q rCDW"

Orbital moment patterns



(a) (b)

"3Q iCDW" "1Q iCDW"